Mathematical Statistics Qualifier Examination
(Part I of the STAT AREA EXAM)
January 25, 2017; 9:00AM – 11:00AM

Name: ________________________ **ID:** ________________ **Signature:** ________________

Instruction: There are 4 problems – you are required to solve them all. Please show detailed work for full credit. This is a close book exam from 9am to 11am. You need to turn in your exam by 11am, and subsequently, receive the questions for your applied statistics exam. Please do NOT use calculator or cell phone. Good luck!

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from a population with pdf

$$f(x|\theta) = \theta^x (1-\theta)^{1-x}, \qquad x = 0 \; or \; 1, \qquad 0 \le \theta \le \frac{1}{2}$$

(a) Find the maximum likelihood estimator and the method of moment estimator for θ.
(b) Find the mean squared errors of each of the estimators.
(c) Which estimator is preferred? Justify your choice.

2. An urn contains one red and one green marble. Draw a marble at random. Toss a fair coin, if the upturned face is head, return the marble to the urn; otherwise put a marble of the other color in the urn. Perform n such drawings (and the corresponding tosses) in succession. Find the limiting distribution of $(X_n - EX_n)/\sqrt{n}$, where X_n is the number of red marbles appearing in the n draws.

3. Let $X_1, X_2, \cdots, X_n$ be a sample taken from a gamma distribution with pdf

$$f(x;\theta) = \theta^2 x e^{-\theta x}, \qquad x > 0, \theta > 0.$$

(a) Prove that this family of distributions has a monotone likelihood ratio.
(b) Suppose that n is large enough so that the Central Limit Theorem can be used. For testing $H_0 : \theta \le \theta_0$ versus $H_a : \theta > \theta_0$ find the acceptance region of the significance level α UMP (uniformly most powerful) test.
(c) Find the $(1-\alpha)$ one-sided confidence interval that results from inverting the test of part (b).

4. Given a random sample $\{X_1, X_2, \cdots, X_n\}$ from a Poisson population with mean λ. Please
(a) Find the maximum likelihood estimator (MLE) of $(1+\lambda)e^{-\lambda}$.
(b) Find the best unbiased estimator of $(1+\lambda)e^{-\lambda}$.
(c) Show that the MLE is a biased estimator of $(1+\lambda)e^{-\lambda}$.

*** That's all, folks! ***